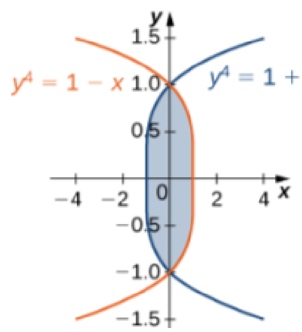
SPRING 2021: MATH 147 QUIZ 6 SOLUTIONS

Each question is worth 5 points. You must provide full details to receive full credit.

Note: These are easier versions of two of the practice problems for Exam 2.

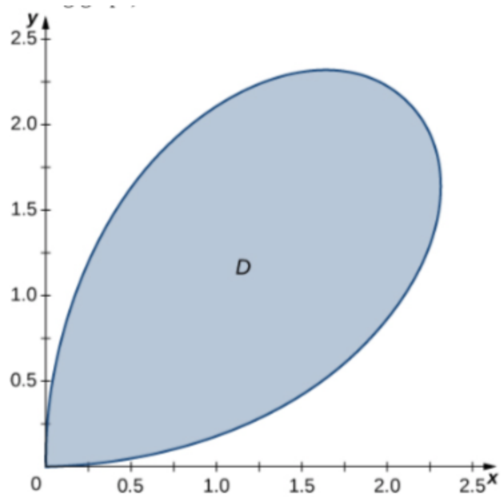
1. The region D is given in the following figure. Evaluate the double integral $\int\int_D (x-y)\ dA$ by using the easier order of integration.



Solution. It is easier to regard D as a region of Type 2. So:

$$\begin{aligned}
\int\int_D (x-y)\ dA &= \int_{-1}^{1}\int_{y^4-1}^{1-y^4} (x-y)\ dx\ dy \\
&= \int_{-1}^{1} (\frac{x^2}{2} - xy)\Big|_{x=y^4-1}^{x=1-y^4}\ dx \\
&= \int_{-1}^{1} \{\frac{(1-y^4)^2}{2} - y + y^5\} - \{\frac{(y^4-1)^2}{2} - y^5 + y\}\ dy \\
&= \int_{-1}^{1} -2y + 2y^5\ dy \\
&= (-y^2 + \frac{2y^6}{6})\Big|_{-1}^{1} \\
&= 0.
\end{aligned}$$

2. Find the area of the region D bounded by the polar curve $r = 3\sin(2\theta)$, where $0 \le \theta \le \frac{\pi}{2}$ (see the following graph). What does this tell you about the area of the region D bounded by $r = 3\sin(2\theta)$, with $0 \le \theta \le 2\pi$?



Solution. The area of D is $\int\int_D \; dA$ which, in polar coordinates gives

$$\begin{aligned}\int_0^{\frac{\pi}{2}}\int_0^{3\sin(2\theta)} r\; dr\; d\theta &= \frac{1}{2}\int_0^{\frac{\pi}{2}} r^2\Big|_0^{r=3\sin(2\theta)}\; d\theta\\ &= \frac{9}{2}\int_0^{\frac{\pi}{2}} \sin^2(2\theta)\; d\theta\\ &= \frac{9}{2}\int_0^{\frac{\pi}{2}} \frac{1}{2}-\frac{1}{2}\sin(4\theta)\\ &= \frac{9}{2}\left(\frac{\theta}{2}+\frac{1}{8}\cos(4\theta)\right)_{\theta=0}^{\theta=\frac{\pi}{2}}\\ &= \frac{9\pi}{8}.\end{aligned}$$

Since D is one petal of a flower with four petals of the same size, the region bounded by $r = 3\sin(2\theta)$ with $0 \le \theta \le 2\pi$ is $4 \cdot \frac{9\pi}{2} = 18\pi$.